



Idaho State
University

Mathematics
and Statistics

Undergraduate
Colloquium

Two Quadrilateral Characteristics

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ABSTRACT. We introduce the cyclic characteristic κ_c and the orthocentric characteristic κ_o of a quadrilateral $\square ABCD$ and discuss their applications. Both κ_c and κ_o are nonnegative real numbers such that $\square ABCD$ is cyclic (resp. orthocentric) if and only if $\kappa_c = 0$ (resp. $\kappa_o = 0$). Moreover, $\square ABCD$ is convex (resp. non-convex) if and only if $\kappa_o > \kappa_c$ (resp. $\kappa_o < \kappa_c$). Let $O_a, O_b, O_c,$ and O_d (resp. $N_a, N_b, N_c,$ and N_d) denote the circumcenters (resp. nine-point centers) of $\triangle BCD, \triangle ACD, \triangle ABD,$ and $\triangle ABC$. When $\square ABCD$ is neither cyclic nor orthocentric, its circumcenter quadrilateral $\square O_a O_b O_c O_d$ and nine-point center quadrilateral $\square N_a N_b N_c N_d$ are similar with $\frac{N_a N_b}{O_a O_b} = \frac{1}{2} \sqrt{\frac{\kappa_o}{\kappa_c}}$. Moreover, the circumcenter quadrilateral of $\square O_a O_b O_c O_d$ and the nine-point center quadrilateral of $\square N_a N_b N_c N_d$ are both homothetic to $\square ABCD$. On the other hand, the circumcenter quadrilateral of $\square N_a N_b N_c N_d$ and the nine-point center quadrilateral of $\square O_a O_b O_c O_d$ are homothetic and congruent. The iteration by constructing the circumcenter or nine-point center quadrilaterals produces a family of quadrilaterals which have the same cyclic and orthocentric characteristics as $\square ABCD$.

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4:00 pm

PS 324

*For colloquium guests, we'll have some cookies!
However, because this talk is in our computer
classroom, we request that no beverages be
brought into the room.)*

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