



# Quadratures and Machine Learning

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In this seminar I will introduce some recent results regarding a new formulation of optimal quadratures and cubatures in terms of a new class of polynomials, the so-called moment-orthogonal polynomials [1]. Such construction allows for approximating unknown functions in terms of their moments which are assured to be exactly reproduced by the function estimator up to a given order. I will show how the approach applies to single-variable functions, interpreted either as black-boxes or as probability distributions. I will demonstrate how the one-dimensional construction maps onto the single-hidden-layer perceptron offering a rare, if not unique, example of implementation of the universal approximation theorem for this kind of neural network architecture. In particular, I will show how the leading contribution corresponds to a Hermite polynomial expansion for a specific choice of the model parameters, suggesting the possibility for proving an  $L^2$  convergence on the real line and on a compact. I will outline the strategy for extending the present construction to higher dimensions, requiring a thermodynamics-inspired approach, i.e., based on the minimization of a free-energy function, enforcing the positivity of the weights for the case of probability distributions.

[1] "Covariant Cubatures & Machine Learning", Matteo Lulli, Emanuele Zappala, Emily S.C. Ching, in preparation.

**Friday, Feb. 14**

**4 - 5 pm**

**PS 307**

**Zoom Meeting ID: 819 2329 9449**

*Light refreshments will be provided.*