Idaho State University

Department of Mathematics & Statistics

Colloquium

Compatibility Conditions for Linear and Nonlinear Elastic Materials and Their Discrete Approximations

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We explore nonlinear and linear prescribed strain equations of plane elasticity and their discretizations. Their compatibility conditions are rarely used and not well understood in discrete models. We study nonlinear and linear continuum models and their discrete approximations focusing on compatibility conditions. It is hoped that counting compatibility conditions in an approximating structure offers a way to measure of resilience and approach understanding cracks and periodic inclusions.

The equation for prescribed deformation tensor in plane elasticity is the equation for a mapping to the Euclidean plane with prescribed pulled back metric. Thus the compatibility condition is that the prescribed metric have vanishing curvature. In linearized elasticity, the corresponding compatibility condition resembles linearized curvature. Suppose that the material domain is triangulated. Imagine that the edges correspond to straight segments forming a truss which approximates the material. The discrete problem is to prescribe the edge lengths and ask if the truss can be realized in the plane. The necessary and sufficient condition is that the induced angles add up to 360 degrees at each interior vertex. A. D. Alexandrov considered this and more general problems of such approximations to metrics with bounded curvature. The linearized discrete problem is to prescribe the infinitesimal elongations of the edges and solve for the infinitesimal displacements of the vertices. The homogeneous case gives the equations of infinitesimal rigidity. The compatibility condition turns out to be that radial elongations balance the elongations going around each interior vertex. This condition implies the continuum condition.

The number of compatibility conditions counts how much the prescribed elongation problem is overdetermined. We explore examples, and derive some results computing the number of compatibility conditions.

This is joint work with Andrej Cherkaev and Predrag Krtolica.

Friday, October 27, 2017
4:00 pm
REND 315

For Colloquium attendees, refreshments will be served in PS 317 at 3:00 pm